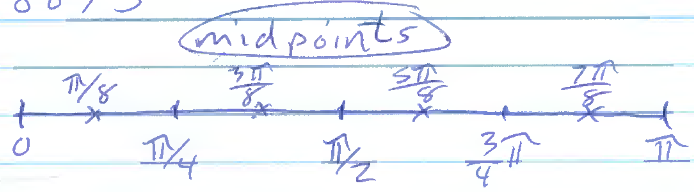
2. From the picture, $f'(x) < 0$ and $f''(x) > 0$, so $R_n < M_n < \int_0^2 f(x)\,dx < T_n < L_n$

a) $0.7811 < 0.8632 < \int_0^2 f(x)\,dx < 0.8675 < 0.9540$

b) $0.8632 < \int_0^2 f(x)\,dx < 0.8675$

6. $\int_0^{\pi} x\cos x\,dx$, $n = 4$



$$M_4 = \Delta x\left[f\left(\tfrac{\pi}{8}\right) + f\left(\tfrac{3\pi}{8}\right) + f\left(\tfrac{5\pi}{8}\right) + f\left(\tfrac{7\pi}{8}\right)\right], \quad \text{where } \Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$$

$$= \boxed{\frac{\pi}{4}\left[\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{3\pi}{8}\cos\frac{3\pi}{8} + \frac{5\pi}{8}\cos\frac{5\pi}{8} + \frac{7\pi}{8}\cos\frac{7\pi}{8}\right]} \approx -1.979072$$

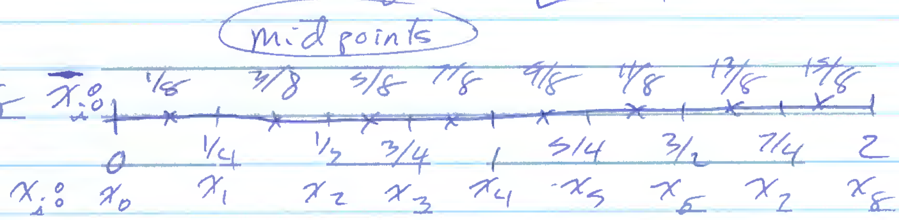
$$S_4 = \frac{\Delta x}{3}\left[f(0) + 4f\left(\tfrac{\pi}{4}\right) + 2f\left(\tfrac{\pi}{2}\right) + 4f\left(\tfrac{3\pi}{4}\right) + f(\pi)\right]$$

$$= \frac{\pi}{12}\left[0 + 4\cdot\frac{\pi}{4}\cos\frac{\pi}{4} + 2\cdot\frac{\pi}{2}\cos\frac{\pi}{2} + 4\cdot\frac{3\pi}{4}\cos\frac{3\pi}{4} + \pi\cos\pi\right]$$

$$\approx -1.985611$$

while $\int_0^{\pi} x\cos x\,dx = \Big[x\sin x + \cos x\Big]_0^{\pi} = \boxed{-2}$

8. $\int_0^2 \frac{1}{1+x^6}\,dx$, $n = 8$



$\Delta x = \frac{2-0}{8} = \frac{1}{4}$

$$T_8 = \frac{1/4}{2}\left[\frac{1}{1+x_0^6} + \frac{2}{1+x_1^6} + \frac{2}{1+x_2^6} + \cdots + \frac{2}{1+x_7^6} + \frac{1}{1+x_8^6}\right] =$$

$$\approx 1.008572$$

$$M_8 = \frac{1}{4}\left[\frac{1}{1+\bar{x}_1^6} + \frac{1}{1+\bar{x}_2^6} + \cdots + \frac{1}{1+\bar{x}_7^6} + \frac{1}{1+\bar{x}_8^6}\right] \approx 1.041109$$

$$S_8 = \frac{1/4}{3}\left[\frac{1}{1+x_0^6} + \frac{4}{1+x_1^6} + \frac{2}{1+x_2^6} + \cdots + \frac{2}{1+x_6^6} + \frac{4}{1+x_7^6} + \frac{1}{1+x_8^6}\right]$$

$$\approx 1.042172$$

34. Distance travelled by runner is equal to $\int_0^5 v(t)\,dt$. Using Simpson's Rule with $n = 10$ and $\Delta t = 0.5$,

$$S_{10} = \frac{1/2}{3}\Big[v(0) + 4\cdot v(0.5) + 2v(1) + 4\cdot v(1.5) + 2v(2) + 4v(2.5) +$$
$$2v(3) + 4v(3.5) + 2v(4) + 4v(4.5) + v(5)\Big]$$

$\cong \boxed{44.735 \text{ meters}}$ where all velocities were drawn from the table